Lecture 2: symmetries in 2HDM

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Outline of lecture 2

1. Symmetries of the potential
2. Symmetries in 2HDM with fermions

Summary from lecture 1

2HDM potential in the bilinear formalism

- the potential depends on $\phi_a^\dagger \phi_b \to r^\mu = (r_0,\, r_i)$:

$$r_0 = \phi_a^\dagger \phi_a\,, \quad r_i = \phi_a^\dagger \sigma^i_{ab} \phi_b\,;$$

- the entire potential with 14 free parameters becomes

$$V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu\,;$$

- basis changes in the ϕ_a space correspond to rotations in r_i space; generalized *CP* transformations (gCP) correspond to improper rotations.

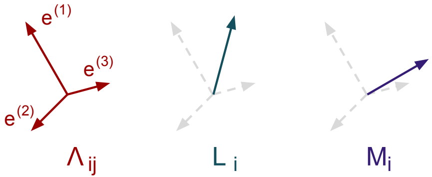
Now, let's study symmetries of the potential.

Structure of the potential

The Higgs potential has three structural terms:

$$V = -M_0 r_0 + \frac{1}{2}\Lambda_{00} r_0^2 + M_i r_i - L_i r_i r_0 + \Lambda_{ij} r_i r_j \,,$$

where $L_i = \Lambda_{0i}$.



Which rotations and reflections leave each of these terms invariant?

Structure of the potential

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$$\Lambda_{ij} r_i r_j = \Lambda_1 e_i^{(1)} e_j^{(1)} + \Lambda_2 e_i^{(2)} e_j^{(2)} + \Lambda_3 e_i^{(3)} e_j^{(3)} \,,$$

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- However, if eigenvalues are degenerate, G_Λ is promoted to a continuous group: $G_\Lambda = O(2) \times \mathbb{Z}_2$ for $\Lambda_1 = \Lambda_2 \neq \Lambda_3$, and $G_\Lambda = O(3)$ for $\Lambda_1 = \Lambda_2 = \Lambda_3$.

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The symmetry group of the potential is

$$G = G_M \cap G_L \cap G_\Lambda .$$

List of symmetries

It is convenient to describe symmetries in the basis in which Λ_{ij} is diagonal:

$$\Lambda_{ij} = \begin{pmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{pmatrix} .$$

In this case, symmetries of $\Lambda_{ij} r_i r_j$ corresponds to independent flips of the axes.

$$r_i = \begin{pmatrix} 2\mathrm{Re}(\phi_1^\dagger \phi_2) \\ 2\mathrm{Im}(\phi_1^\dagger \phi_2) \\ (\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2) \end{pmatrix} .$$

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For example, the *CP* transformation $\phi_a \to \phi_a^*$ corresponds to reflection of r_2. The transformation suggested in lecture 1

$$\phi_1 \to \phi_2^* , \quad \phi_2 \to \phi_1^* ,$$

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Questions

Q2.1: what's the effect of the diagonalizing basis change on the original parameters λ_i?

Q2.2: find the gCP transformation which realizes axis flip $r_1 \to -r_1$.

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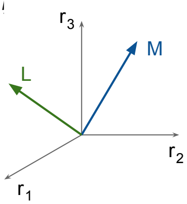
- no alignment: M_i, L_i is oriented generically: $M_i = (M_1, M_2, M_3)$, and all components are non-zero $\to$ no symmetry remains.



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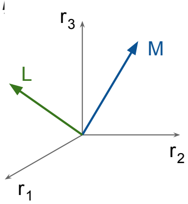
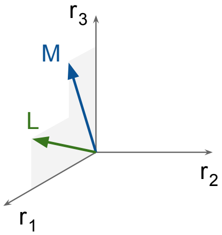


- minimal alignment: both M_i and L_i are orthogonal to an eigenvector of $\Lambda_{ij} \to M_i = (M_1, 0, M_3)$, $L_i = (L_1, 0, L_3)$, and the model is invariant under r_2 flip $\to$ *CP*-conserving 2HDM.



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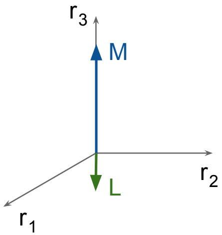
- complete alignment: M_i and L_i are parallel to each other and to an eigenvector of Λ_{ij}. In the diagonal-Λ_{ij} basis, the model can always be brought to $M_i = (0, 0, M_3)$, $L_i = (0, 0, L_3)$, and is invariant under independent flips of r_1 and of r_2 → $\mathbb{Z}_2$-invariant 2HDM.



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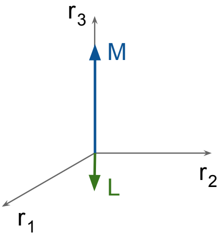
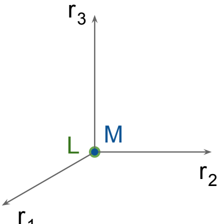


- maximal discrete symmetry: $M_i = L_i = 0 \rightarrow$ maximal CP-conservation, the model has three intependent gCP transformations.



In the last two cases, if eigenvalues of Λ_{ij} are degenerate, the symmetry group can become continuous.

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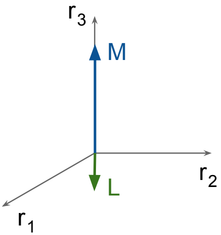

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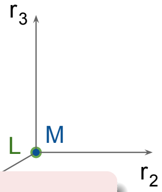

In the ... ry group can become continuous.

All these symmetries can be hidden.
No problem, the geometric conditions are basis-invariant!

List of symmetries

The final list: the scalar potential of 2HDM can have only one of the following six symmetry classes:

groups in r_i-space:	$\mathbb{Z}_2$	$(\mathbb{Z}_2)^2$	$(\mathbb{Z}_3)^3$	$O(2)$	$O(2)\times\mathbb{Z}_2$	$O(3)$
standard notation:	$CP1$	$\mathbb{Z}_2$	$CP2$	$U(1)$	$CP3$	$U(2)$

The CP1, CP2, CP3 classes of gCP symmetries are defined as $\phi_a \to U_{ab}\phi_b^*$, where

$$U^{(CP1)} = 1\,, \quad U^{(CP2)} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,, \quad U^{(CP3)} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\,.$$

For more details, see *[Ferreira, Haber, Silva, 2009] and [REVIEW]*.

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For m

Question

Q2.3: prove that CP2 transformation is equivalent to $(\mathbb{Z}_2)^3$

Further issues

- The above geometric criteria for existence of symmetries can be cast in purely algebraic conditions in terms of basis-independent invariants. This is a rather simple linear algebraic exercise, in huge contrast with the tensoris invariants built of Y_{ab} and $Z_{ab,cd}$.

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> In short, the geometric bilinear formalism answers practically all symmetry-related questions for the scalar sector of 2HDM.

Yukawa interactions in SM

In the SM with 1 generation of quarks, where $Q_L = (u_L, d_L)$ is a EW-doublet, d_R is EW-singlet, the EWSB gives $\langle\phi\rangle = (0, v/\sqrt{2})^T$, and the quark masses arise via the Higgs vev:

$$\mathcal{L}_Y = -f_d(\bar{Q}_L \phi d_R + \bar{d}_R \phi^\dagger Q_L) \to -\frac{f_d v}{\sqrt{2}}(\bar{d}_L d_R + \bar{d}_R d_L) = -m_d \bar{d} d \,.$$

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For the up-quark, we use $\tilde{\phi} = i\sigma_2 \phi^*$ with $\langle\tilde{\phi}\rangle = (v/\sqrt{2},\, 0)^T$:

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With three generations Q_{Li}, d_{Ri}, u_{Ri}, the complication is marginal:

$$-\mathcal{L}_Y = \bar{Q}_{Li}\Gamma_{ij}\phi d_{Rj} + \bar{Q}_{Li}\Delta_{ij}\tilde{\phi} u_{Rj} + h.c. \to (M_d)_{ij}\bar{d}_{Li} d_{Rj} + (M_u)_{ij}\bar{u}_{Li} u_{Rj} + h.c.$$

Diagonalization of M_d and M_u must involve different left field transformations: $V_{dL} \neq V_{uL}$, otherwise we'd have the trivial CKM matrix.

FCNC

Although M_d and M_u are complicated, we know that their diagonalization automatically makes $H\bar{q}q$ coupling diagonal. This is because $\phi \to (H+v)/\sqrt{2}$, so H couples to quarks in the same way as v $\to$ in SM, the Higgs exchange does not induce FCNC (flavor-changing neutral currents).

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In 2HDM, there is a danger of inducing FCNC:

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Any bSM model, in order to be phenomenologically viable, must keep FCNC suppressed.

We need to make sure that FCNC ≈ 0 holds in 2HDM.

$\mathbb{Z}_2$ symmetry in Yukawa interactions

The classical way to eliminate tree-level FCNC in 2HDM is to impose $\mathbb{Z}_2$ symmetry on the Yukawa interaction $\to$ natural flavor conservation.

The most studied cases are:

- Type-I 2HDM: all quarks couple to ϕ_2 ($\Gamma_1 = \Delta_1 = 0$) $\to \mathcal{L}_Y$ is invariant under $\phi_1 \to -\phi_1$.
- Type-II 2HDM: all up-quarks couple to ϕ_2, all down-quark couple to ϕ_1 ($\Gamma_2 = \Delta_1 = 0$) $\to \mathcal{L}_Y$ is invariant under $\phi_1 \to -\phi_1$, $d_R \to -d_R$. This structure arises in MSSM.

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Phenomenology of Type-I and Type-II 2HDM has been studied in much detail, see *[REVIEW]* and works *[Aoki, Kanemura, Shimura, Yagyu, 2009]*, *[Celis, Ilisie, Pich, 2013]*, *[Crivellin, Kokulu, Greub, 2013]*, *[Eberhardt, Nierste, Wiebusch, 2013]*.

$\mathbb{Z}_2$ symmetry in Yukawa interactions

If the $\mathbb{Z}_2$ symmetry of the Yukawa terms holds in the scalar potential, then we get the entire model symmetric under $\mathbb{Z}_2$. The scalar potential is rather simple and can be worked out explicitly. And it lacks CP-violation in the scalar sector.

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The meaning of "soft" violation is that at very small distances (= in processes with large virtualities $\gg v^2$), the effects of $\phi_1 \leftrightarrow \phi_2$ mixing is suppressed, and the $\mathbb{Z}_2$ symmetry is asymptotically restored. This leaves a possibility to link $\mathbb{Z}_2$ to a deeper model in the UV limit.

Inert model

A special version of Type-I 2HDM with the exact $\mathbb{Z}_2$ symmetry known as inert doublet model *[Ma, 2006], [Barbieri, Hall, Rychkov, 2006], [Lopez Honorez, Nezri, Oliver, Tytgat, 2007]*:

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- ϕ_2 does not couple to any fermions,
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Then, the $\mathbb{Z}_2$ symmetry is unbroken, and ϕ_2 becomes inert: it does not contribute to masses of W and Z, it does not contribute to masses of fermions, and the lightest physical Higgses from ϕ_2, be it H, A, or $H^\pm$, is stable. It becomes a natural scalar dark matter candidate.

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$$\Gamma_1\phi_1 + \Gamma_2\phi_2 \to \Gamma'_1\phi'_1 + (\Gamma'_2 = 0)\phi'_2 = \Gamma'_1\phi'_1\,,$$

and *another* Higgs basis change $\phi \to \phi''$ which decouples from u-quarks:

$$\Delta_1\tilde{\phi}_1 + \Delta_2\tilde{\phi}_2 \to \Delta''_1\tilde{\phi}''_1 + (\Delta''_2 = 0)\tilde{\phi}''_2 = \Delta''_1\tilde{\phi}''_1\,.$$

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$$\Delta_1\tilde{\phi}_1 + \Delta_2\tilde{\phi}_2 \to \Delta''_1\tilde{\phi}''_1 + (\Delta''_2 = 0)\tilde{\phi}''_2 = \Delta''_1\tilde{\phi}''_1\,.$$

Proportionality is a basis-independent statement that some doublet decouples from quarks with equal quantum numbers.

Suppressing FCNC without $\mathbb{Z}_2$ symmetry

A more recent and more relaxed suggestion is aligned 2HDM: $\Gamma_1 \propto \Gamma_2$, $\Delta_1 \propto \Delta_2$ *[Pich, Tuzon, 2009]*. It means that there exists a Higgs basis change $\phi \to \phi'$ which decouples one Higgs doublet from *d*-quarks

$$\Gamma_1\phi_1 + \Gamma_2\phi_2 \to \Gamma'_1\phi'_1 + (\Gamma'_2 = 0)\phi'_2 = \Gamma'_1\phi'_1\,,$$

and *another* Higgs basis change $\phi \to \phi''$ which decouples from *u*-quarks:

$$\Delta_1\tilde\phi_1 + \Delta_2\tilde\phi_2 \to \Delta''_1\tilde\phi''_1 + (\Delta''_2 = 0)\tilde\phi''_2 = \Delta''_1\tilde\phi''_1\,.$$

Proportionality is a basis-independent statement that some doublet decou

Question

Q2.4: in aligned 2HDM, the bases $\{\phi'_1, \phi'_2\}$ and $\{\phi''_1, \phi''_2\}$ can be different. Find how they are related in Type-I and Type-II 2HDMs, which can be viewed as very particular cases of aligned 2HDM.

Suppressing FCNC without $\mathbb{Z}_2$ symmetry

Yet another possibility is to allow for non-zero Γ_k and Δ_k but assume that they possess some natural structure. It is convenient to switch to the Higgs basis $(\phi_1, \phi_2) \to (H_1, H_2)$ such that $\langle H_1^0 \rangle = v/\sqrt{2}$, $\langle H_2^0 \rangle = 0$.

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In the Higgs basis, the Yukawa matrices become:

$$\Gamma_1 \phi_1 + \Gamma_2 \phi_2 \to X_d H_1 + Y_d H_2\,, \quad \Delta_1 \tilde{\phi}_1 + \Delta_2 \tilde{\phi}_2 \to X_u \tilde{H}_1 + Y_u \tilde{H}_2\,.$$

H_1 generates fermion masses and $H_1 \bar{q} q$ vertices, H_2 produces only $H_2 \bar{q} q$ vertices.

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If matrices $Y = 0$ (Type-I, II), no FCNC occur. If matrices $Y \propto X$ (aligned 2HDM), no FCNC occur. $Y \neq 0$ and not $\propto X$ can lead to FCNC, but certain structures of Y's can lead to FCNC which are very small; a couple of famous examples: *[Cheng, Sher, 1987]*, *[Branco, Grimus, Lavoura, 1996]*.

In general, generating various textures for Yukawa matrices from different principles with naturally small FCNC is an active topic, see more examples in *[REVIEW]*.

Is there life beyond $\mathbb{Z}_2$?

All discrete symmetries we discussed so far were made of $\mathbb{Z}_2$, either as a Higgs-family $\mathbb{Z}_2$ or a gCP transformation. Is it possible to construct a 2HDM with other discrete symmetry groups?

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If we limit ourselves to the scalar sector of 2HDM, the answer is no: we already have the full list of symmetry groups, and there is nothing but powers of $\mathbb{Z}_2$. It means that even if we impose by force another discrete symmetry group on 2HDM scalars, say $\mathbb{Z}_3$, we will unavoidably get a model with continuous symmetry. This is not what we ask for.

Is there life beyond $\mathbb{Z}_2$?

But we can ask for symmetries which act not only on Higgses but also on fermions.

The simplest example: rephasing of Higgses and quarks:

$$\phi_a \to \phi_a^e i\theta_a\,, \quad Q_{Li} \to Q_{Li} e^{i\alpha_i}\,, \quad d_{Rj} \to d_{Rj} e^{i\beta_j}\,, \quad u_{Rk} \to u_{Rk} e^{i\gamma_k}\,.$$

Which discrete rephasing groups can be achieved in 2HDM with quarks?

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Which discrete rephasing groups can be achieved in 2HDM with quarks?

This question was investigated in *[Ferreira, Silva, 2011]* with the following results. It is possible to construct 2HDM with quarks with the $\mathbb{Z}_3$ symmetry. However, if you try to impose any higher $\mathbb{Z}_n$, $n > 3$, you automatically obtain a model with an accidental continuous symmetry. Thus, $\mathbb{Z}_2$ and $\mathbb{Z}_3$ are the only discrete abelian groups available in 2HDM with quarks.

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Question

Q2.5: A non-abelian group G can have various abelian subgroups. Suppose all of its abelian subgroups are isomorphic to $\mathbb{Z}_2$ or to $\mathbb{Z}_3$. Find G.

Lecture 2 Summary

- Within scalar sector of 2HDM, one can answer virtually all symmetry related questions with the bilinear formalism. There are six classes of symmetries, their conditions are known, their breaking and phenomenology have been studied.
- Extending symmetries to fermion sector opens up additional possibilities. Even the simplest group $\mathbb{Z}_2$ can be implements in various ways, with different phenomenological consequences, and there arise other discrete symmetry groups beyond $\mathbb{Z}_2$.